

TUM-HEP 789/11
January 14, 2011

Rare Kaon Decays

EMMANUEL STAMOU*

*Institute for Advanced Study, Technische Universität München,
Lichtenbergstraße 2a, D-85748 Garching, Germany*

*Physik-Department, Technische Universität München,
James-Franck-Straße, D-85748 Garching, Germany*

Rare Kaon decays provide sensitive probes of short distance physics. Among the most promising modes are the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ modes due to their theoretical cleanness and the dedicated experiments, NA62 and KOTO. The key to the success of New Physics searches in these modes is the precision in the standard model prediction. We review the status and the recent progress of their standard model prediction, and also discuss the predictions and New Physics sensitivities of the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ and $K_L \rightarrow \mu^+ \mu^-$ decays.

PRESENTED AT

CKM 2010

the 6th International Workshop on the CKM Unitarity
Triangle
University of Warwick, UK, 6-10 September 2010

*estamou@ph.tum.de

1 Introduction

Within the standard model (SM), short distance (SD) physics accounts for approximately 90%, 40% and 40% of the branching fractions of the rare Kaon decays $K \rightarrow \pi \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \ell^+ \ell^-$ and $K_L \rightarrow \mu^+ \mu^-$, respectively. These decays are all governed by a flavour changing neutral current (FCNC) transition and therefore loop induced within the SM. Loop induced processes are special in the sense that heavy new particles may contribute to the decay amplitudes on the same footing as SM particles. Thus, they represent ideal probes for searching and disentangling New Physics (NP). To succeed in isolating NP contributions it is essential to have an accurate SM prediction with good theoretical control over both short- and long-distance (LD) effects.

In the following we briefly discuss the general structure of rare Kaon decays and to which extend each decay channel can reveal information about degrees of freedom beyond the SM. We review the SM prediction of the above-mentioned modes and also present the recent electroweak (EW) calculation on the top-quark contribution to the two $K \rightarrow \pi \nu \bar{\nu}$ decays.

2 Structure of Rare Kaon Decays

Rare Kaon decays proceed within the SM via the quark-level transition $s \rightarrow d$ through the box and γ, Z -penguin diagrams of Fig. 1. However, at the Kaons' mass scale of approximately 500 MeV the physics is best described by an appropriate effective field theory (EFT) in which heavy particles like top-quark, W- and Z-bosons are no longer dynamical degrees of freedom. A master amplitude then applies to all meson decays [1]:

$$\mathcal{A}_{\text{decay}} = \sum_i \langle Q_i \rangle V_{\text{CKM}}^i F_i. \quad (1)$$

$\langle Q_i \rangle$ are matrix elements of effective operators, that cannot be calculated perturbatively, V_{CKM}^i comprise products of CKM matrix elements and F_i are the so-called Wilson coefficients, effective coupling constants that parametrise effects of SD physics. The sum in Eq. (1) extends over all possible operators Q_i generated by the SM or a given NP model and similarly over contributions from different up-type quarks in the loops of Fig. 1. The EFT approach allows to separate SD from LD physics and therefore to treat these contributions independently. While the matrix elements of semileptonic decays can be extracted from experimental data, the Wilson coefficients depend on the high-energy model and can be calculated perturbatively within the SM.

Two points should be stressed regarding Eq. (1). First, F_i are process independent universal loop functions within the SM, meaning that they are the same for B-, D-

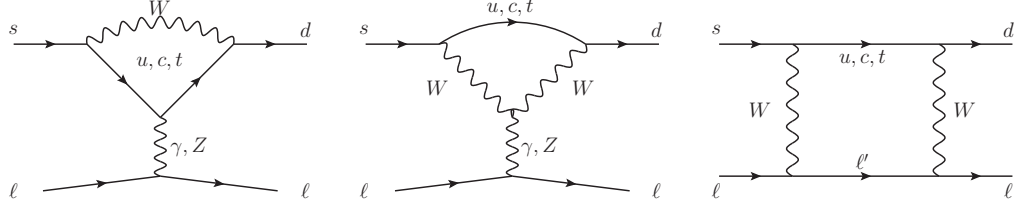


Figure 1: γ, Z penguins and W boxes contributing to the $s \rightarrow d$ transition at the one-loop level. In the case $l \equiv \nu$ no γ -penguin contributes.

and K-meson decays. The resulting correlation of observables in the SM tests the universality of the loop functions in a model independent way (see talk by D. Straub [2]). Secondly, NP can not only change the Wilson coefficients, but can also generate additional operators contributing to the decay amplitude. This would signal a departure from Minimal Flavour Violation [3]. Both NP effects can affect the branching ratios of rare Kaon decays.

The master amplitude of Eq. (1) also incorporates the relative magnitude of each contribution. Its size depends on both the CKM elements, conveniently parametrised in terms of $\lambda = |V_{us}| \approx 0.22$, and on the loop functions F_i 's. Within the SM, low-energy contributions are suppressed with respect to the top contribution due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [4]. When SD photon penguins contribute, this suppression is only logarithmic, but in their absence (e.g. in the $K \rightarrow \pi \nu \bar{\nu}$ decays) the loop functions are proportional to m_{top}^2 , m_{charm}^2 , or Λ_{QCD}^2 for the up-quark, and a hard quadratic GIM is at play.

In addition, the CKM factor of the top-quark contribution $\lambda_t = V_{ts}^* V_{td}$ is proportional to λ^5 and therefore highly suppressed. This strong suppression, absent in B and D decays, can imply a significant charm-contribution depending on the mode, but also renders rare Kaon decays sensitive to even very small deviations from the CKM picture of CP violation.

3 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The weak SM effective Hamiltonian for the two $K \rightarrow \pi \nu \bar{\nu}$ decays reads [5]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_{l=e,\mu,\tau} (\lambda_c X^l + \lambda_t X_t) (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_{lL} \gamma^\mu \nu_{lL}) + h.c. \quad (2)$$

and involves to a good approximation only one effective operator below $\mu = \mu_c$.

The LD part of both decays is known with high precision. The matrix elements are extracted from $K_{\ell 3}$ decays including isospin-breaking ($\kappa_\nu^{+,L}$) and long-distance QED

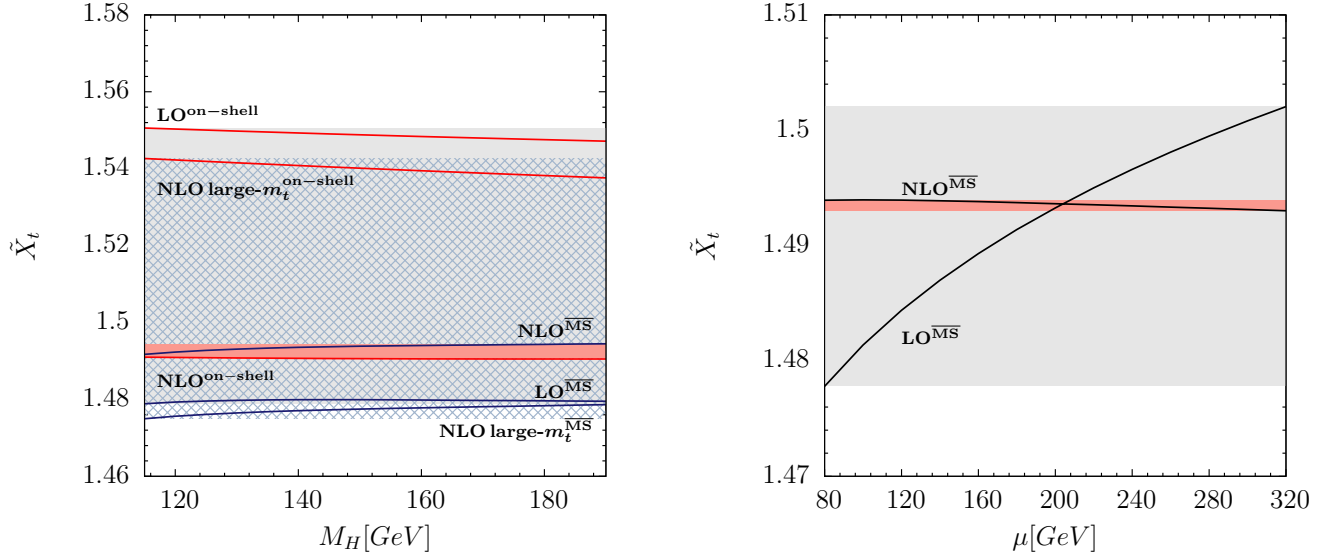


Figure 2: Left: comparison of the top-quark Wilson coefficient in the $\overline{\text{MS}}$ and on-shell schemes at LO, NLO large- m_t and NLO. The grey solid band, hashed band, and solid red band represent the estimated uncertainties of LO, NLO large- m_t , and NLO result as a function of the Higgs mass, respectively. Right: the uncertainty due to EW renormalisation scale at LO and NLO. Again the grey and red band represent the estimated uncertainty before and after the NLO calculation, respectively.

effects [6], whereas also effects from light quarks and higher dimensional operators ($\delta P_{c,u}$) have been estimated using chiral perturbation theory (ChPT) [7].

Regarding the small uncertainties from LD effects, higher order corrections on the SD parts become essential for a precise SM prediction. The SD charm contribution, denoted by $P_c = \frac{1}{\lambda^4} (\frac{2}{3} X^e + \frac{1}{3} X^\tau)$, is negligible for $K_L \rightarrow \pi^0 \nu \bar{\nu}$, but not for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, where it amounts to approximately 30% of its branching ratio. So far P_c is known up to next-to-next-to-leading order (NNLO) in QCD [8] and also the next-to-leading order (NLO) EW corrections have been calculated [9]. NLO QCD corrections [5, 10] are known for the top-quark contribution, but until recently the NLO EW corrections were only known in the large- m_t limit [11]. However, these were known to poorly approximate the full EW corrections (see Fig. 2) [11]. Therefore, the renormalisation scheme of the EW input parameters $\alpha, \sin \theta_W, M_W, M_t$, appearing at LO in Eq. (2), was not clear and accounted for an approximately 2% uncertainty in X_t , which scales quadratically in the uncertainty of the branching ratios.

Our recent two-loop calculation of the full NLO EW corrections on X_t [12] substantially reduced the renormalisation scheme uncertainty to approximately 0.2% and also removed the remaining scale dependence. The comparison of LO and NLO results in the $\overline{\text{MS}}$ and on-shell scheme is illustrated in Fig. 2 together with the remaining

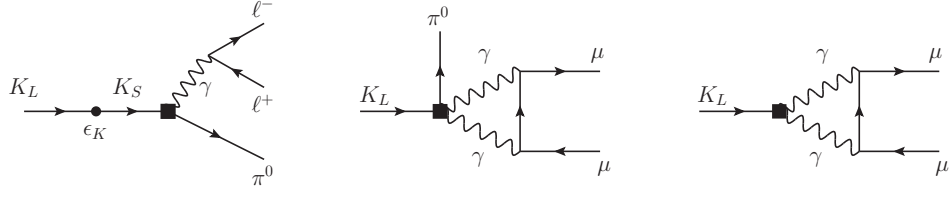


Figure 3: γ - and $\gamma\gamma$ - penguins contributing to the LD part of the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ and $K_L \rightarrow \mu^+ \mu^-$ branching ratios.

scale dependence.

The theoretical predictions and the current experimental values for the branching ratios then read:

$$\begin{aligned} \text{BR}_{K^+ \rightarrow \pi^+ \nu \bar{\nu}}^{\text{theo}} &= (7.81_{-0.71}^{+0.80} \pm 0.29) \times 10^{-11} [12] & \text{BR}_{K^+ \rightarrow \pi^+ \nu \bar{\nu}}^{\text{exp}} &= (1.73_{-1.05}^{+1.15}) \times 10^{-10} [13] \\ \text{BR}_{K_L \rightarrow \pi^0 \nu \bar{\nu}}^{\text{theo}} &= (2.43_{-0.37}^{+0.40} \pm 0.06) \times 10^{-11} [12] & \text{BR}_{K_L \rightarrow \pi^0 \nu \bar{\nu}}^{\text{exp}} &< 6.7 \times 10^{-8} [14] \end{aligned}$$

The first error in the theory prediction is parametric, while the second summarises the remaining theory uncertainty. For the charged mode the detailed main parametric contributions are ($V_{cb} : 56\%$, $\bar{\rho} : 21\%$, $m_c : 8\%$, $m_t : 6\%$, $\bar{\eta} : 4\%$, $\alpha_s : 3\%$, $\sin^2 \theta_W : 1\%$), while the main theoretic contributions are ($\delta P_{c,u} : 46\%$, $X_t(\text{QCD}) : 24\%$, $P_c : 20\%$, $\kappa_\nu^+ : 7\%$, $X_t(\text{EW}) : 3\%$), respectively. Similarly for the neutral mode, the parametric uncertainties are ($V_{cb} : 54\%$, $\bar{\eta} : 39\%$, $m_t : 6\%$) and the theoretical are ($X_t(\text{QCD}) : 73\%$, $\kappa_\nu^L : 18\%$, $X_t(\text{EW}) : 8\%$, $\delta P_{c,u} : 1\%$), respectively.

4 $K_L \rightarrow \pi^0 \ell^+ \ell^-$

In the two $K_L \rightarrow \pi^0 \ell^+ \ell^-$ modes three different contributions compete in magnitude: direct- CP -violating (DCPV), indirect- CP -violating (ICPV) and CP -conserving (CPC) contribution. Long distance effects are much larger here. Still, the importance of these decay channels lies in their sensitivity to SD contributions from more than one operator. Let us now look at the three distinct contributions.

DCPV: high-energy top and charm contributions generate both vector, $Q_{7V} = (\bar{s}d)_V(\bar{\ell}\ell)_V$, and axial-vector, $Q_{7A} = (\bar{s}d)_V(\bar{\ell}\ell)_A$, dimension-six operators. Their Wilson coefficients are known to NLO QCD [3] and their matrix elements are also extracted with a few per mil precision from $K_{\ell 3}$ decays [6]. One difference between the electronic and the muonic channels is the suppression of the electronic axial-vector matrix element due to the smallness of the electron mass.

ICPV: in this case the K_L decays through its small CP even component via the γ -penguin in Fig. 3. The size of this smaller component is described by the parameter ϵ_K . We can therefore relate this contribution to the decay $K_S(\approx K_1) \rightarrow \pi^0 \ell^+ \ell^-$ using

ChPT. The light meson loops are subleading and the amplitude is dominated by the chiral counterterm a_S [15], whose 20% uncertainty completely dominates the error for the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ rates [16]. Although the absolute value of a_S has been extracted from NA48 measurements, its sign remains unknown. Therefore, it is still an open question whether the ICPV contribution interferes constructively or destructively with the DCPV contribution. Apart from a more precise $K_S \rightarrow \pi^0 \ell^+ \ell^-$ measurement, the sign of a_S can also be determined from the measurement of the integrated forward-backward asymmetry A_{FB}^ℓ in $K_L \rightarrow \pi^0 \mu^+ \mu^-$ [16].

CPC: is a purely LD effect described by the $\gamma\gamma$ -penguin ChPT diagram in Fig. 3. The contribution is helicity suppressed for the electronic mode, a further difference between the two modes. For the muonic mode the meson loops are finite at $\mathcal{O}(p^4)$, while higher order effects of $\mathcal{O}(p^6)$ have been partially estimated from the $K_L \rightarrow \pi^0 \gamma\gamma$ ratio, reducing the uncertainty to approximately 30% [17].

The current SM predictions and experimental values for the branching ratios for the two decays then read:

$$\begin{aligned} \text{BR}_{K_L \rightarrow \pi^0 \mu^+ \mu^-}^{\text{theo}} &= 1.41_{-0.26}^{+0.28} (0.95_{-0.21}^{+0.22}) \times 10^{-11} [16] & \text{BR}_{K_L \rightarrow \pi^0 \mu^+ \mu^-}^{\text{exp}} &< 3.8 \times 10^{-10} [18] \\ \text{BR}_{K_L \rightarrow \pi^0 e^+ e^-}^{\text{theo}} &= 3.54_{-0.85}^{+0.98} (1.56_{-0.49}^{+0.62}) \times 10^{-11} [16] & \text{BR}_{K_L \rightarrow \pi^0 e^+ e^-}^{\text{exp}} &< 2.8 \times 10^{-10} [19] \end{aligned}$$

Theory predictions correspond to constructive interference of DCPV with ICPV contribution, while predictions in brackets to destructive interference.

5 $K_L \rightarrow \mu^+ \mu^-$

$K_L \rightarrow \mu^+ \mu^-$ is a CP conserving decay, whose SD sensitivity is obscured by large LD effects. The top contribution is known up to NLO QCD [20] and the charm contribution up to NNLO QCD [21]. Indirect CP violation is negligible in this channel. However, in contrast to the $K_L \rightarrow \pi^0 \mu^+ \mu^-$ case, the LD $\gamma\gamma$ -penguin is not finite (Fig. 3). The absorptive part of the $\gamma\gamma$ -loop, extracted from $K_L \rightarrow \gamma\gamma$, accounts to approximately 98% of the measured branching ratio, $\text{BR}_{K_L \rightarrow \mu^+ \mu^-}^{\text{exp}} = 6.84(11) \times 10^{-9}$ [22], while dispersive part of the $\gamma\gamma$ -contribution is divergent in ChPT and can therefore be estimated from $K_L \rightarrow \gamma^* \gamma^*$ only with a large uncertainty [23]. On top of these difficulties, the $\gamma\gamma$ -contribution interferes with an unknown sign with the SD contribution, which further increases the theoretical uncertainty of the channel. Therefore, the interesting SD part of the decay mode cannot be extracted accurately. Nevertheless, useful NP constraints can be derived using its measured branching ratio.

6 New Physics and Outlook

The theoretical cleanness of the $K \rightarrow \pi \nu \bar{\nu}$ modes promotes both to excellent probes of NP, especially in view of the dedicated experiments, NA62 at CERN and KOTO at

JPARC, which aim at measuring the charged and neutral mode, respectively, with an expected accuracy of 15%. These decays have therefore been studied extensively in models beyond the SM. A summary of predictions in different models is presented in [24]. This analysis points out the possibility of large effects in both decay modes and also illustrates the correlation between the predictions of the two decays in a large class of NP models.

NP effects on the branching ratios of the $K_L \rightarrow \pi^0 \ell^+ \ell^-$ modes have also been considered, in a model-independent way. A measurement of the decays could test contributions from operators not generated by the SM. Effects from scalar, pseudoscalar, tensor, and pseudotensor operators, generated by NP with or without helicity suppression, have been studied and can be disentangled by a more precise measurement of both modes [16].

To conclude, rare Kaon decays are very clean and sensitive probes of NP. The observation of deviations from the SM by the dedicated experiments would be a clear NP signal. Also, the use of both predictions and measurements as constraints of NP can shed light on NP patterns and the structure of flavour violation.

Acknowledgements

I would like to thank the conveners of working group III and the organisers of CKM2010 for the interesting time in Warwick, Joachim Brod and Martin Gorbahn for their support and the fruitful collaboration, and Andrzej Buras for comments on the manuscript. This research was supported by the DFG cluster of excellence "Origin and Structure of the Universe".

References

- [1] G. Buchalla, A. J. Buras and M. E. Lautenbacher. Rev. Mod. Phys. **68**:1125, 1996. [hep-ph/9512380](#).
- [2] D. M. Straub 2010. [1012.3893](#).
- [3] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini. Nucl. Phys. **B592**:55, 2001. [hep-ph/0007313](#).
- [4] S. L. Glashow, J. Iliopoulos and L. Maiani. Phys. Rev. **D2**:1285, 1970.
- [5] G. Buchalla and A. J. Buras. Nucl. Phys. **B548**:309, 1999. [hep-ph/9901288](#).
- [6] F. Mescia and C. Smith. Phys. Rev. **D76**:034017, 2007. [0705.2025](#).

- [7] G. Isidori, F. Mescia and C. Smith. Nucl. Phys. **B718**:319, 2005. [hep-ph/0503107](#).
- [8] A. J. Buras, M. Gorbahn, U. Haisch and U. Nierste. JHEP **11**:002, 2006. [hep-ph/0603079](#).
- [9] J. Brod and M. Gorbahn. Phys. Rev. **D78**:034006, 2008. [0805.4119](#).
- [10] M. Misiak and J. Urban. Phys. Lett. **B451**:161, 1999. [hep-ph/9901278](#).
- [11] G. Buchalla and A. J. Buras. Phys. Rev. **D57**:216, 1998. [hep-ph/9707243](#).
- [12] J. Brod, M. Gorbahn and E. Stamou 2010. [1009.0947](#).
- [13] A. V. Artamonov *et al.* Phys. Rev. Lett. **101**:191802, 2008. [0808.2459](#).
- [14] J. K. Ahn *et al.* Phys. Rev. **D81**:072004, 2010. [0911.4789](#).
- [15] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles. JHEP **08**:004, 1998. [hep-ph/9808289](#).
- [16] F. Mescia, C. Smith and S. Trine. JHEP **08**:088, 2006. [hep-ph/0606081](#).
- [17] G. Isidori, C. Smith and R. Unterdorfer. Eur. Phys. J. **C36**:57, 2004. [hep-ph/0404127](#).
- [18] A. Alavi-Harati *et al.* Phys. Rev. Lett. **84**:5279, 2000. [hep-ex/0001006](#).
- [19] A. Alavi-Harati *et al.* Phys. Rev. Lett. **93**:021805, 2004. [hep-ex/0309072](#).
- [20] G. Buchalla and A. J. Buras. Nucl. Phys. **B400**:225, 1993.
- [21] M. Gorbahn and U. Haisch. Phys. Rev. Lett. **97**:122002, 2006. [hep-ph/0605203](#).
- [22] K. Nakamura *et al.* J. Phys. **G37**:075021, 2010.
- [23] G. Isidori and R. Unterdorfer. JHEP **01**:009, 2004. [hep-ph/0311084](#).
- [24] F. Mescia and C. Smith URL <http://www.lnf.infn.it/wg/vus/content/Krare.html>.